United Kingdom
Mathematics Trust

Mathematical Olympiad for Girls

Thursday 5 October 2017

Organised by the United Kingdom Mathematics Trust

Instructions

1. Do not turn over the page until told to do so.
2. Time allowed: $2\frac{1}{2}$ hours.
3. Each question carries 10 marks. Full marks require clearly written solutions — not just answers — including complete proofs of any assertions you may make.

   Marks awarded will depend on the clarity of your mathematical presentation. Work in rough first, and then write up your best attempt.
4. Partial marks may be awarded for good ideas, so try to hand in everything that documents your thinking on the problem — the more clearly written the better.

   However, one complete solution will gain more credit than several unfinished attempts.
5. The use of rulers and compasses is allowed, but calculators and protractors are forbidden.
6. Start each question on a fresh sheet of paper. Write on one side of the paper only.

   On each sheet of working write the number of the question in the top left-hand corner and your name, initials and school in the top right-hand corner.
7. Complete the cover sheet provided and attach it to the front of your script, followed by your solutions in question number order.
8. Staple all the pages neatly together in the top left hand corner.
9. To accommodate candidates sitting in other time zones, please do not discuss the paper on the internet until 08:00 BST on Friday 6 October.

Enquiries about the Mathematical Olympiad for Girls should be sent to:

MOG, UK Mathematics Trust, School of Mathematics,
University of Leeds, Leeds LS2 9JT

☎ 0113 343 2339   enquiry@ukmt.org.uk   www.ukmt.org.uk

1. All the digits 1 to 9 are to be placed in the circles in Figure 1, one in each, so that the total of the numbers in any line of four circles is the same. In the example shown in Figure 2, the total is equal to 20.

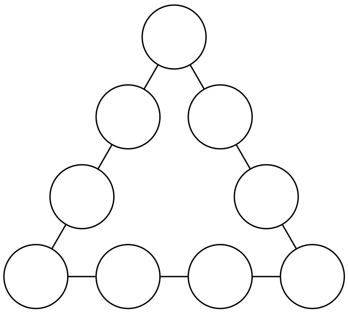

Figure 1

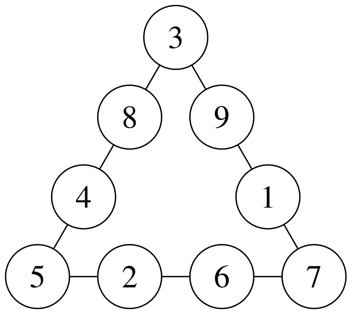

Figure 2

Prove that if the total T is possible then the total $40 - T$ is possible.

2. A positive integer is said to be *jiggly* if it has four digits, all non-zero, and no matter how you arrange those digits you always obtain a multiple of 12.

How many jiggly positive integers are there?

3. Four different points A, B, C and D lie on the curve with equation $y = x^2$.

Prove that $ABCD$ is *never* a parallelogram.

4. Let n be an odd integer greater than 3 and let $M = n^2 + 2n - 7$.

Prove that, for all such n, at least four different positive integers (excluding 1 and M) divide M exactly.

5. Claire and Stuart play a game called *Nifty Nines*:

(i) they take turns to choose one number at a time, with Claire choosing first;
(ii) numbers can only be chosen from the integers 1 to 5 inclusive;
(iii) the game ends when n numbers have been chosen (repetitions are permitted).

Stuart wins the game if the sum of the chosen numbers is a multiple of 9, otherwise Claire wins.

Find all values of n for which Claire can ensure a win, whatever Stuart's choices were. You must prove that you have found them all.