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Mathematical Olympiad for Girls 2025

Teachers are encouraged to distribute copies of this report to candidates.

Markers' report

Introduction

For many students, and possibly some teachers, this is the first experience of attempting a Maths Olympiad paper. It may therefore be useful to understand how these papers are marked, as students may be disappointed to receive a small number of marks for a problem they thought they had almost solved.

Most questions in Olympiad papers are marked using what we call the '0+/10−' principle. This means that the markers first read the whole write-up and decide whether the student has a viable strategy to solve the problem. It may be that there are some mistakes or small gaps in their reasoning, but if those could be relatively easily filled in then this response is marked in the '10−' regime, with usually up to three marks being taken away for gaps and mistakes. Common examples of small gaps are algebraic or arithmetical errors (provided they don't change the nature of the argument), missing one of several cases in a counting question, or lack of geometrical reasons when calculating angles.

If, on the other hand, the student has only started to explore the problem and has only made some useful observations, but does not have a strategy to generalise or prove them, then the script is marked in the '0+' regime. Up to three marks may be available for spotting a pattern or trying an idea which, if progressed further, could lead to a solution. Examples of this in the present paper would be: in Question 4, understanding how the ideas from parts (a) and (b) can be applied in part (c); or in Question 5, trying to find a strategy for B that involves some sort of mirroring or restoring of symmetry. Notice that these all involve a substantial engagement with the problem, rather than just looking at some examples or special cases. Teachers should therefore reiterate to students that scoring even one or two marks on any of these questions is a real achievement.

The Mathematical Olympiad for Girls is slightly different from other Maths Olympiads in that questions are broken down into several parts. Most of the time, the final part is the "main question" and the first part (or parts) are intended to suggest some useful results or good approaches to the problem. The reason for structuring the paper in this way is that the setters know that many of the candidates are not experienced in olympiad mathematics, and the hope is that by giving these pointers, we enable them to engage with the questions even if they are not familiar with standard olympiad techniques or "tricks". A useful hint is to read the whole question first and try to understand how the early parts may be helpful in solving the main problem.

The present paper also contains some answer-only questions, which nevertheless require a similar way of thinking to the usual olympiad questions. These questions are designed so that partial marks can be awarded to incorrect answers which show good progress towards the correct solution.

General comments

This year saw the introduction of the Mathematical Competition for Girls (MCG) to complement the MOG and allow even more students to take part. The MCG paper contained four of the same questions, but all in the answer only format. Consequently the number of MOG entries has decreased, although the number of participating schools continues to rise.

The questions this year required students to explore and think creatively. Questions 1, 2, 3 and 5 certainly allowed for playing with small cases to get an idea of the general structure. It was great to see that almost all candidates engaged with all five questions and scored marks on the early parts. The markers were also impressed with the number of write ups of a very high quality.

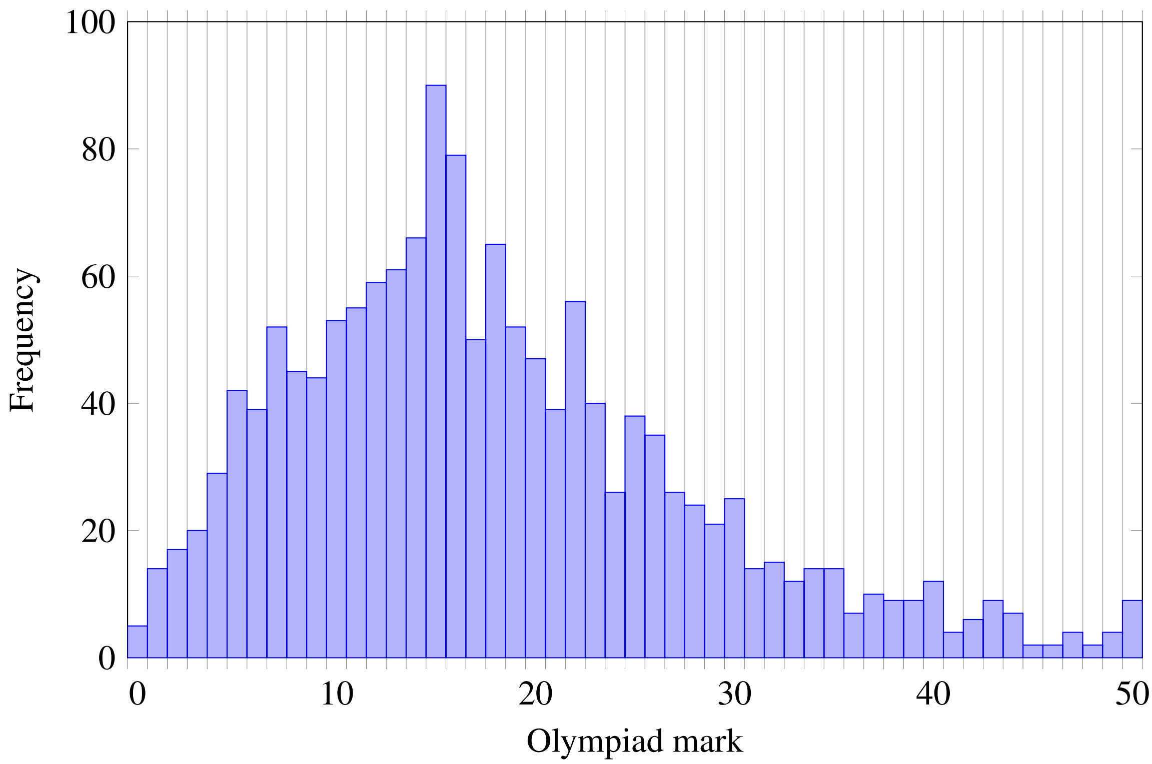
The knowledge required to engage with the problems was again relatively basic. However, careful thinking was needed in order to reach the answer in the answer-only questions and to form a valid argument in all three full-solution ones.

It is unfortunately often the case that students think that they have solved a problem but only receive two or three marks. The main example this year was Question 5, where many students came up with strategies for part (c) which only work under the assumption that player A plays in a particular way. In all games questions, it is important to understand that a winning strategy must work regardless of whether the other player plays "well" or "badly". This is a very important lesson to learn for any student hoping to progress further in Maths Olympiads.

Mark distribution

The MOG 2025 paper was marked by a team of Hugh Ainsley, Margaret Anthony, Ruth Barber, Naomi Bazlov, Phill Beckett, Robin Bhattacharyya, Malcolm Blake, Abdellatif Charafi, Raka Chattopadhyay, Lin Chen, Andrea Chlebikova, John Cullen, Gareth Davies, Carl Dolby, Chris Eagle, Ben Fairfax, Thomas Frith, Chris Garton, Ben Gillott, Rebekah Glaze, Edward Godfrey, Anthony Goncharov, Aleksander Goodier, Aditya Gupta, Jon Hart, Kelsey Hewitt, Alexander Hurst, Michael Illing, Ahmed Ittihad Hasib, Elad Kalif, Thomas Kavanagh, Kit Kilgour, Audrey Kueh, Sida Li, Elsa Lin, Aleksander Lishkov, Thomas Lowe, Eleanor MacGillivray, Oliver Murray, Joseph Myers, Jamie Nevill, Calvin Newn, Huyen Ngoc Pham, Peter Price, Heerpal Sahota, James Sarkies, Harcharan Singh Sidhu, Geoff Smith, Stephen Tate, Amit Srivastava, Velian Velikov, Kasia Warburton, Emma Wheeler, Yue Wu and Lingde Yang.

Following the restriction of entries to a maximum of four per school, we received responses from 1479 candidates. The mean mark was 17.9 and the median was 16.



The award thresholds were: 14 for Merit, 24 for Distinction and 44 for a Book Prize.

Question 1

This question requires answers only.

Consider the number $M = 99\ldots99$ which consists of several digit nines. A single division sign is placed between two adjacent digits of M and the resulting calculation is evaluated to produce a whole number N.

(a) In the case when M has nine digits,

(i) How many possible values can N take? [1 mark]

(ii) How many digits does the smallest possible value of N have? [2 marks]

(b) In the case when M has 2025 digits,

(i) How many possible values can N take? [3 marks]

(ii) How many digits does the smallest possible value of N have? [4 marks]

SOLUTION

(See the official solutions document.)

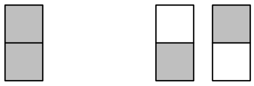
MARKERS' COMMENTS

This was found a challenging opening question and it split the field somewhat, with around a quarter of the candidates scoring close to full marks but around a third unable to score more than two. Nevertheless, it seems that many candidates engaged with the question and gained some insight into what is going on. Of those who were close to full marks, most were able to answer part (b)(ii) – finding that the answer had 676 digits. Counting the number of possible values of N correctly (as 14) was tricky, and it was possible to receive partial marks for answers of 15, 7 or 8, which suggests some confusion about which factors of 2025 can be used to obtain valid values of N.

Question 2

This question requires full written solutions.

Tom has a large supply of two types of dominoes, Type A and Type B. Type B dominoes can be rotated 180° so that the grey square is on top.



type A type B

Tom wants to select three dominoes and place them next to each other to create a 2×3 rectangle (so the dominoes remain vertical, as shown above). He wants both top and bottom rows of his rectangle to contain at least one square of each colour.

(a) (i) How many sequences can Tom make which contain exactly one Type A domino? [2 marks]

(ii) How many sequences of three dominoes can he make in total? [2 marks]

(b) Each of the white squares on Type B dominoes has a whole number between 1 and 6 (inclusive) written on it. Grey squares have no numbers on them. There are several copies of each numbered Type B domino.

(The numbers do not change when a Type B domino is rotated; for example a 6 does *not* become a 9.)

Tom wants to select and arrange three dominoes as before (with both top and bottom rows containing at least one square of each colour, and using dominoes of either type), but now he also wants the numbers on the top row to add up to 6 and the numbers on the bottom row to add up to 6.

In how many ways can he do this? [6 marks]

SOLUTION

(See the official solutions document.)

MARKERS' COMMENTS

This proved to be the most successful question, with over half the candidates scoring close to full marks. We saw some very clear reasoning and well structured solutions, and very few examples of calculations with no attempt at explanations.

The key was to organise your working in a clear manner and not get lost in which cases you have already considered to allow you to count all of them carefully. The question was structured in a way which suggested splitting the counting into smaller sub-cases. It was encouraging to see that a good number of scripts realised that using (a)(i) in (a)(ii) and then both of these in (b) was the way to proceed, and hence split up their working into the cases of having 1 or 0 type A dominoes, which made both the working out and the marking clear.

Interestingly, a significant minority got the wrong answers in part (a), often because of forgetting the rules and placing the two Type B dominoes into not-allowed positions, but then found a different method to answer part (b) correctly. Thinking about the link between the two parts of the question can often help make corrections to early parts.

Some scripts misunderstood what the question was getting at; reading the question carefully a couple of times over would have definitely helped these. For (a)(i) in particular, a lot of scripts tried to use arguments about placing the type A domino and then the type B dominoes. Sometimes, these arguments got to the wrong answer – it is a good idea to at least draw out the examples to try and see that this matches the answer that you get, as this can help convince you that your argument was correct, especially when the number of correct examples is quite small.

In part (b), lots of scripts realised that when a row has only one white square, it has to contain a 6, and the only other case was when a row would have two white squares and so contain a pair of positive integers summing to 6. In part (b) common errors included forgetting that when having two white squares with 3s in one row, the order in which they appear doesn't matter but also that when having two white squares with (1,5) or (2,4) in them, the order in which they appear does matter!

Question 3

This question requires answers only.

(a) Positive (non-zero) whole numbers a and b satisfy $(a+b)(a-b)=45$. Find all possible values of a. [2 marks]

(b) Priya and Rhia each create a sequence of positive integers.

Priya starts with 1000 and adds consecutive odd numbers, so that her sequence begins 1000, 1001, 1004, 1009.

Rhia starts with 3025 and also adds consecutive odd numbers.

(i) How many numbers (including 3025) appear in both sequences? [4 marks]

(ii) Find the **second** smallest number that appears in both sequences. [4 marks]

SOLUTION

(See the official solutions document.)

MARKERS' COMMENTS

Many candidates engaged successfully with this question, with almost all providing at least some correct answers to part (a), although just over 10% got close to full marks.

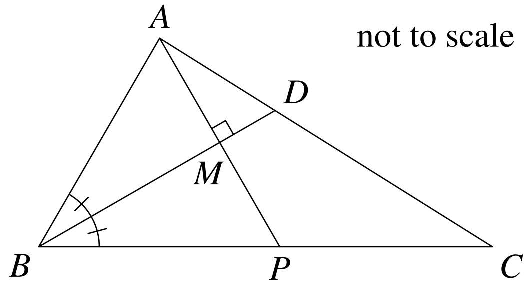
It was common to see only two of the three values of a in part (a), mostly missing $a = 23$, which suggests that they may have forgotten the factorisation 1×45. A small but significant number of candidates did not read the question carefully, providing answers such as values of b or values of a^2, or including negative values of a.

Part (b)(ii) was answered reasonably well, with many candidates giving either the correct answer of 3061 or one of the larger common terms. Part (b)(i) proved more challenging, although the most common incorrect answers, 7 and 15, suggest that candidates were looking for factors of 2025, and these answers scored partial marks.

Question 4

This question requires full written solutions.

The diagram shows triangle ABC with side lengths $AB = 85$, $BC = 160$ and $CA = 103$ units. The bisector of angle ABC intersects the side AC at point D. The line through A perpendicular to BD intersects BD at M and BC at P.



(a) Prove that M is the midpoint of AP. [2 marks]

(b) Find the length of PC, justifying your answer. [1 mark]

(c) The line through A perpendicular to the bisector of angle ACB intersects it at N. Find the length of MN. [7 marks]

Solution

(See the official solutions document.)

Markers' comments

As is often the case with geometry questions, a large number of candidates did not offer much of an attempt. Those who did were generally able to score at least two of the three marks available for parts (a) and (b).

The vast majority of candidates realised that triangles ABM and PBM are congruent, or that triangle ABP is isosceles, and were able to use this to find the length of PC and score the mark for part (b). Slightly fewer managed to score both marks in part (a). Those who explained why the two triangles are congruent were generally successful; however, those who tried to explain why ABP is an isosceles triangle often provided flawed arguments.

It is indeed true that if an angle bisector is perpendicular to the opposite side then the triangle is isosceles, but this needed to be stated clearly rather than mentioned alongside various other properties of isosceles triangles. In particular, it is important to realise that this is a different statement from "if a triangle is isosceles then the angle bisector at the apex is perpendicular to the base", which can not be used to prove that a triangle *is* isosceles. The correct flow of logic in "if-then" statements is extremely important in mathematics, with geometry proofs offering a very good illustration.

It was common to see arguments referring to the "kite" $ABPD$ without explaining clearly how we know that the shape is indeed a kite. One correct argument is that the reflection in the line BD maps A to P, because of the two equal angles and the fact that AP is perpendicular to BD. Hence $ABPD$ has DB as a line of symmetry, which is the definition of a kite.

In part (c), most candidates who noticed the link to the first two parts were able to solve the problem completely. A small number managed to either find the length of BQ (where Q is the intersection of AN and BC) or conclude that N is the midpoint of AQ, but were not able to use tho two together to complete the question.

Around 12% of the candidates solved the problem, with almost all of them scoring full marks with clearly explained solutions. Many of the candidates were probably not familiar with the crucial result needed for this part: if M and N are midpoints of AP and AQ then MN is parallel to, and half the length of, PQ. In this case, MN is called the *midline* of triangle APQ. Although this result can be quoted as a fact in Olympiad solutions, many candidates proved it successfully by referring to similar triangles. However, the flow of logic is important again: triangles AMN and APQ are similar because $AP : AM = AQ : AN = 2 : 1$ *and* they share the angle at A; then the similarity implies that $PQ : MN = 2 : 1$ as well.

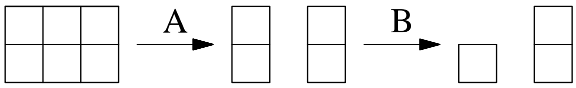
Question 5

This question requires full written solutions.

Two players, A and B, play a game with a set of square tiles. To start, the tiles are arranged in a rectangle. A turn consists of removing a row or column of tiles from one rectangle (possibly splitting it into two). A rectangle consisting of only one row or one column may be removed in a single turn.

Players take turns, with Player A going first. The player removing the final tile(s) wins.

(a) In this part, the game starts with a 2×3 rectangle. The diagram below shows the first two moves. Player A starts by removing the middle column.



Complete the diagram to show one way in which Player A can win the game.

[1 mark]

(b) In this part, the game starts with a 20×25 rectangle. Describe a strategy for Player A which guarantees that they will win the game, no matter what Player B does on their turns. [2 marks]

(c) In this part, the game starts with a 20×24 rectangle. Determine which player has a winning strategy. [7 marks]

You must describe what the winning player should do on each turn, which may depend on what the other player does, and explain why these moves always lead to a win.

SOLUTION

(See the official solutions document.)

MARKERS' COMMENTS

The vast majority of candidates attempted this difficult question.

Part (a) was generally answered well. A reasonable proportion of candidates scored at least one mark for part (b) by picking up on the hint that removing the central column was a useful way to start, but there was much confusion about what to do next. Part (c) spurred some remarkable creativity, even if very few candidates succeeded in describing a valid strategy.

Why was this problem so troublesome? The main issue seems to be that there are many plausible strategies that "feel" correct but that are, in fact, hopeless; and candidates generally do not write with enough precision to enable them to see exactly where those bogus strategies fail.

We will now describe a few of the non-strategies encountered during the marking of part (c). Remember, it's a 20×24 grid, A starts, and – as most candidates guessed correctly – B should win.

The first is: on each move, B reverts the configuration to one in which A is facing an even number of tiles. Loose motivation: zero is even, and if A faces no tiles, B wins. Although this clearly fails if A is facing a single 1×2 tile, it's still plausible that this particular endgame would never arise and that the strategy might work if you start with a grid as large as 20×24.

We know that this is a non-strategy because we can construct games in which B obeys the prescribed rules, but hands A a winning position. Here's one example; hopefully the notation is clear:

```
A => (13,24) + (6,24)
B => (13,24) + (6,12) + (6,11)
A => (13,22) + (13,1) + (6,12) + (6,11)
B => (13,21) + (13,1) + (6,12) + (6,11) => ... A wins
```

The winning condition for the current player turns out to be that they are facing an odd number of rectangles that have four odd sides *and/or* an odd number of rectangles that have two odd sides. The lone even × odd (6×11) region is what hands the win to A in this case.

The second non-strategy is that B reverts the configuration to one in which the sum of the heights of the disjoint rectangles, and the sum of the widths, are both even. The motivation is that if A and B both only ever removed rows and columns from the edge of the (single) rectangle, then B would in fact win. However, a successful strategy must not rely on the opponent moving in a particular way. Here's a counterexample when A's moves are not so constrained:

```
A => (20,17) + (20,6)
B => (20,13) + (20,3) + (20,6)         : R=60, C=22
A => (20,13) + (19,3) + (20,6)
B => (20,13) + (19,3) + (3,6) + (16,6) : R=58, C=28
```

The third non-strategy is overenthusiastic bisection. Inspired by removal of the middle column in part (a), many candidates proceeded to make B remove the middle column of rectangles at every opportunity, reverting to mirroring A's move across the previous bisecting line when no further bisections are available. There's really no reason why this should work (what if A has adopted the same strategy?), and indeed it doesn't:

```
A => (4,24) + (15,24)
B => (4,24) + (7,24) + (7,24)
A => (4,24) + (7,22) + (7,1) + (7,24)
B => (4,24) + (7,22) + (7,1) + (3,24) + (3,24)
```

Ill-motivated though it may seem at first glance, variants on overenthusiastic bisection abounded, probably because the actual strategy (the first of the official solutions, which markers dubbed "twin-making") is so hard to describe accurately.

The final strategy worth mentioning is mirroring. This is well motivated by the general principle that if B can continually impose symmetry upon A, then B wins. *Rotational* symmetry works. *Mirror* symmetry about either of the two midlines of the full grid usually doesn't unless B's responses to A's midline-crossing moves are chosen very carefully.

To appreciate the issue with mirroring, consider the strategy whereby B mirrors A's moves in "the mirror line" parallel to the line removed in A's turn. Which mirror line? Suppose A had taken a row. The mirror line can't be the horizontal midline of the current rectangle (fails if it was odd-by-odd and A took the middle row). The mirror line can't be the midline of the full grid (fails if the grid is split into three horizontal bands consisting of 1, 1 and 3 rectangles, then in consecutive moves A takes the bottom row of two of the three lower rectangles). There seems to be no consistent choice of mirror line that works in this particular case, nor was one offered by the candidates.

There were around seventy scripts that managed to avoid the traps described above and presented a working strategy. The most successful were often quite concise, homing-in on the precise notion of symmetry or "evenness" that the winning player should preserve, and tracking the state of the game efficiently to arrive at the conclusion in part (c) that B wins. Less precise arguments received fewer marks; arguments that appeared to be non-strategies scored very little.