UK Maths Trust

Mathematical Olympiad for Girls

Thursday 25 September 2025

Organised by the United Kingdom Mathematics Trust

supported by Jane Street Overleaf

Instructions

1. Do not turn over the page until told to do so.
2. Time allowed: $2\frac{1}{2}$ hours.
3. Each question carries 10 marks.
4. Questions 1 and 3 require answers only. The spaces for answers are clearly indicated on the answer sheets.
5. Questions 2, 4 and 5 require full written explanations. If your solution involves calculations, equations, tables, etc., explain where these come from and how you are using them. Explain how the steps of your solution link together, and give full proofs of assertions that you make. Answers alone will gain few marks (if any).
6. Partial marks may be awarded for good ideas, so try to hand in everything that documents your thinking on the problem — the more clearly written the better.
   However, one complete solution will gain more credit than several unfinished attempts.
7. Earlier questions tend to be easier. Questions have multiple parts. Often earlier parts introduce results or ideas useful in solving later parts of the problem. You should read the whole question before you start solving the first part.
8. The use of rulers and compasses is allowed, but calculators and protractors are forbidden.
9. You may use rough paper to note down your ideas, but you should write up your solution on the answer sheet provided for each question.
10. Start each question on an official master answer sheet that has a QR code on it.
    You may use additional sheets (blank or lined paper only). On each additional sheet please write the number of the question in the top left-hand corner, followed by the QR code digits following the ':' symbol. Please do not write your name or initials on additional sheets.
11. Write on one side of the paper only.
12. Arrange your answer sheets in question order before they are collected. Please remove blank answer sheets.
13. To accommodate candidates sitting in other time zones, please do not discuss the paper on the internet until 8am BST on Saturday 27th September, when the solutions video will be released at ukmt.org.uk/video-solutions-list.

Enquiries about the Mathematical Olympiad for Girls should be sent to:

challenges@ukmt.org.uk

www.ukmt.org.uk

Throughout this paper, you may use the fact that 2025 has 15 factors and its prime factorisation is $3^4 \times 5^2$.

1. *This question requires answers only.*

Consider the number $M = 99 \ldots 99$ which consists of several digit nines. A single division sign is placed between two adjacent digits of M and the resulting calculation is evaluated to produce a whole number N.

(a) In the case when M has nine digits,

(i) How many possible values can N take? [1 mark]

(ii) How many digits does the smallest possible value of N have? [2 marks]

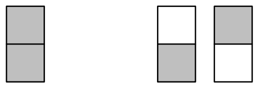
(b) In the case when M has 2025 digits,

(i) How many possible values can N take? [3 marks]

(ii) How many digits does the smallest possible value of N have? [4 marks]

2. *This question requires full written solutions.*

Tom has a large supply of two types of dominoes, Type A and Type B. Type B dominoes can be rotated 180° so that the grey square is on top.



type A type B

Tom wants to select three dominoes and place them next to each other to create a 2×3 rectangle (so the dominoes remain vertical, as shown above). He wants both top and bottom rows of his rectangle to contain at least one square of each colour.

(a) (i) How many sequences can Tom make which contain exactly one Type A domino? [2 marks]

(ii) How many sequences of three dominoes can he make in total? [2 marks]

(b) Each of the white squares on Type B dominoes has a whole number between 1 and 6 (inclusive) written on it. Grey squares have no numbers on them. There are several copies of each numbered Type B domino.

(The numbers do not change when a Type B domino is rotated; for example a 6 does *not* become a 9.)

Tom wants to select and arrange three dominoes as before (with both top and bottom rows containing at least one square of each colour, and using dominoes of either type), but now he also wants the numbers on the top row to add up to 6 and the numbers on the bottom row to add up to 6.

In how many ways can he do this? [6 marks]

3. *This question requires answers only.*

(a) Positive (non-zero) whole numbers a and b satisfy $(a+b)(a-b) = 45$. Find all possible values of a. [2 marks]

(b) Priya and Rhia each create a sequence of positive integers.

Priya starts with 1000 and adds consecutive odd numbers, so that her sequence begins 1000, 1001, 1004, 1009.

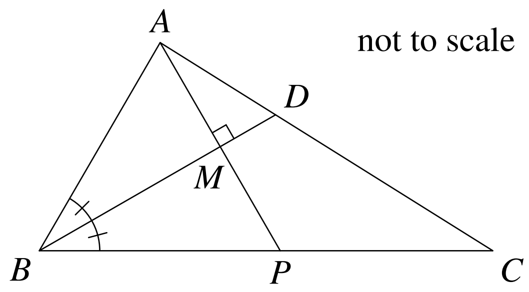
Rhia starts with 3025 and also adds consecutive odd numbers.

(i) How many numbers (including 3025) appear in both sequences? [4 marks]

(ii) Find the **second** smallest number that appears in both sequences. [4 marks]

4. *This question requires full written solutions.*

The diagram shows triangle ABC with side lengths $AB = 85$, $BC = 160$ and $CA = 103$ units. The bisector of angle ABC intersects the side AC at point D. The line through A perpendicular to BD intersects BD at M and BC at P.



(a) Prove that M is the midpoint of AP. [2 marks]

(b) Find the length of PC, justifying your answer. [1 mark]

(c) The line through A perpendicular to the bisector of angle ACB intersects it at N.

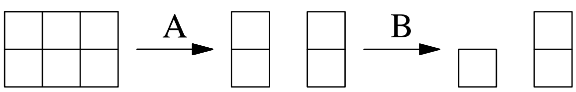
Find the length of MN. [7 marks]

5. *This question requires full written solutions.*

Two players, A and B, play a game with a set of square tiles. To start, the tiles are arranged in a rectangle. A turn consists of removing a row or column of tiles from one rectangle (possibly splitting it into two). A rectangle consisting of only one row or one column may be removed in a single turn.

Players take turns, with Player A going first. The player removing the final tile(s) wins.

(a) In this part, the game starts with a 2×3 rectangle. The diagram below shows the first two moves. Player A starts by removing the middle column.



Complete the diagram to show one way in which Player A can win the game.

[1 mark]

(b) In this part, the game starts with a 20×25 rectangle. Describe a strategy for Player A which guarantees that they will win the game, no matter what Player B does on their turns. [2 marks]

(c) In this part, the game starts with a 20×24 rectangle. Determine which player has a winning strategy. [7 marks]

You must describe what the winning player should do on each turn, which may depend on what the other player does, and explain why these moves always lead to a win.