UK Maths Trust

Mathematical Olympiad for Girls

Tuesday 22 September 2026

Organised by the United Kingdom Mathematics Trust

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Instructions

1. Do not turn over the page until told to do so.
2. Time allowed: $2\frac{1}{2}$ hours.
3. Each question carries 10 marks.
4. Questions 1 and 3 require answers only. The spaces for answers are clearly indicated on the answer sheets.
5. Questions 2, 4 and 5 require full written explanations. If your solution involves calculations, equations, tables, etc., explain where these come from and how you are using them. Explain how the steps of your solution link together, and give full proofs of assertions that you make. Answers alone will gain few marks (if any).
6. Partial marks may be awarded for good ideas, so try to hand in everything that documents your thinking on the problem — the more clearly written the better.
   However, one complete solution will gain more credit than several unfinished attempts.
7. Earlier questions tend to be easier. Questions have multiple parts. Often earlier parts introduce results or ideas useful in solving later parts of the problem.
8. The use of rulers and compasses is allowed, but calculators and protractors are forbidden.
9. You may use rough paper to note down your ideas, but you should write up your solution on the answer sheet provided for each question.
10. Start each question on an official master answer sheet that has a QR code on it.
    You may use additional sheets (blank or lined paper only). On each additional sheet please write the number of the question in the top left-hand corner, followed by the QR code digits following the ':' symbol. Please do not write your name or initials on additional sheets.
11. Write on one side of the paper only.
12. Arrange your answer sheets in question order before they are collected. Please remove blank answer sheets.
13. To accommodate candidates sitting in other time zones, please do not discuss the paper on the internet until 08:00 BST on Thursday 24th September 2026.

Enquiries about the Mathematical Olympiad for Girls should be sent to:

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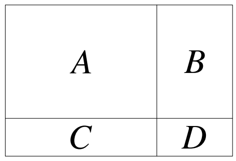
www.ukmt.org.uk

1. *This question requires answers only.*

A rectangular apartment is divided into four rectangular rooms, by two straight interior walls which are parallel to the exterior walls. The rooms have areas A, B, C and D metres squared.



(a) Find the value of D when $A = 8$, $B = 4$ and $C = 12$. [1 mark]

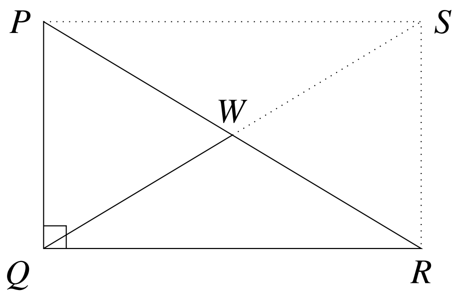
(b) Find the value of C when $A = 7.5$, $B = 6.5$ and $D = 26$. [2 marks]

(c) Find an expression for C in terms of A, B and D. [2 marks]

(d) In this part of the question, A, B, C and D have the values found in part (a). Additionally, the length and the width of each room are whole numbers.

Let P be the perimeter of the apartment. Find all possible values of P. [5 marks]

2. (a) Triangle PQR has a right angle at Q and W is the midpoint of the side PR. Use the diagram below to show that $QW = RW$. [2 marks]



(b) Triangle ABC has a right angle at B and $AB < BC$. Let M be the midpoint of AC. Let D be the reflection of C in the line BM (extended beyond M).

Given that angle DAC is $x°$ find, in terms of x, the size of angle BAC. [8 marks]

You must show all your calculations and say what geometrical result(s) you used at each step.

3. *This question requires answers only.*

(a) You have three red and four blue cubes. The cubes of the same colour are identical.

This part of the question is about the number of ways to distribute all seven cubes among some boxes, following these rules:

- A box may be empty (so it is possible to put all seven cubes in one box).
- The boxes are considered different (so putting all seven cubes in the first box counts as a different solution from putting all seven cubes in the second box).

(i) How many ways are there to distribute all seven cubes among two boxes? [2 marks]

(ii) How many ways are there to distribute all seven cubes among three boxes, so that no colour appears in all three boxes? [3 marks]

(b) The *factorial* of a positive integer n is defined as $n! = 1 \times 2 \times 3 \times \cdots \times n$. For example, $3! = 1 \times 2 \times 3 = 6$.

In how many ways can we write $10! = abc$, where a, b, c are positive integers, such that there is no factor greater than one which is common to all three of them?

Reorderings of a, b, c count as different solutions. For example, $120 \times 54 \times 560$ is not allowed because all three numbers share a common factor of 2, while $640 \times 162 \times 35$ is allowed and counts as a different solution from $162 \times 640 \times 35$. [5 marks]

4. This question is about finding winning strategies in two-player games. We say that a player has a *winning strategy* if they can win the game no matter what moves the other player makes.

(a) In the first game, Players A and B start with a pile of 100 counters and take turns to remove either 2 or 3 counters from the pile. The winner is the player who removes the final counter. If only one counter remains, the result is a draw.

Player A goes first. Describe the winning strategy for Player B, and explain why this strategy guarantees a win. [2 marks]

(b) Players A and B are now playing a different game. They start with a pile of 700 counters, and take turns to remove 1, 3 or 4 counters from the pile. The winner is the player who removes the final counter. (If there are only two counters left, the current player must remove one counter.)

Player A goes first.

(i) Show that Player B can ensure that, at the end of their first turn, there are either 693 or 695 counters remaining. [1 mark]

(ii) Show that Player B can win if, at the end of one of their turns, there are 7 or 9 counters remaining. [1 mark]

(iii) Determine which player has a winning strategy.

You should

- Describe how the winning player should select their moves, which may depend on the other player's moves.
- Explain why this strategy guarantees that this player will take the final counter.

[6 marks]

5. A cuboid has side lengths $a - x$, a and $a + x$, where a, x are positive integers with $a > x$. The main body diagonal of the cuboid (connecting one vertex to the opposite vertex, passing through the centre of the cuboid) has length d.

This question asks you to prove that it is impossible for d to be an integer.

(a) Write down and simplify an equation relating a, x and d. [1 mark]

(b) Find all possible remainders that a square number can give when divided by 8. Hence prove that it is not possible for d to be an odd integer. [3 marks]

(c) Now suppose that d is an even integer.

(i) Prove that a and x must also be even. [1 mark]

(ii) Let $a = 2a_1$, $x = 2x_1$ and $d = 2d_1$. Find and simplify an equation relating a_1, x_1 and d_1. [1 mark]

(d) Complete the proof that it is impossible for d to be an integer. [4 marks]

